Intelligent Explorations of the String Theory Landscape

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Work in collaboration with
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mainly based on 2204.08073

- The success of string phenomenology crucially depends on overcoming a number of technical hurdles.
- Machine learning (AI, more generally) is no panacea, but can alleviate a lot of pain.

Summary

- Quasi-topological formulae for line bundle cohomology on CY3. These formulae provide an instantaneous method for the computation of particle spectra in heterotic string models. ML techniques have played an important role in this discovery.

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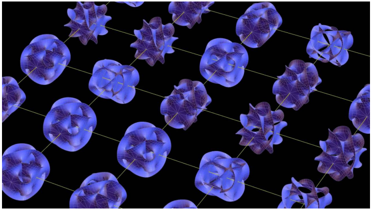
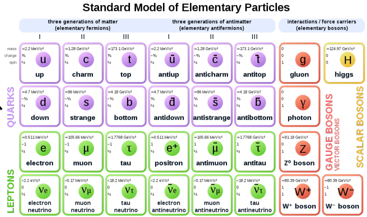
- Quasi-topological formulae for line bundle cohomology on CY3. These formulae provide an instantaneous method for the computation of particle spectra in heterotic string models. ML techniques have played an important role in this discovery.
- Searching for physically relevant models within ultra-large spaces of possible compactifications is feasible with heuristic methods such genetic algorithms (GAs) and reinforcement learning (RL).
- The progress made in the computation of Calabi-Yau metrics, hermitian bundle metrics and harmonic forms using ML techniques is fast and it will soon be possible to compute physical Yukawa couplings in string theory models.



Top-down approach

Bottom-up approach



Three approaches to string phenomenology

- Top-down approach: too many possibilities
  - 10^{500} consistent type IIB flux compactifications [Douglas 2003]
  - $10^{272,000}$ F-theory flux compactifications [Taylor, Wang 2015]
  - 10^{723} heterotic models with the exact chiral spectrum of the Standard Model [AC, He, Lukas 2018]
  - 10^{15} F-theory models with the exact chiral spectrum of the Standard Model [Cvetič, Halverson, Lin, Liu, Tian 2019]
- Bottom-up approach: technically too difficult
- New half-way approach based on AI methods

Model building steps

- correct low-energy gauge group
- three families of quarks and leptons

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- stabilise the moduli
- break supersymmetry

$E_8 \times E_8$ Heterotic String Compactifications

- the earliest proposal for string phenomenology
- low energy limit: 10-dim. $\mathcal{N} = 1$ supergravity coupled to $E_8 \times E_8$ SYM theory; built-in gauge and gravitational degrees of freedom
- dimensional reduction: need a 6d compact manifold X; to preserve $\mathcal{N} = 1$ supersymmetry in 4d, X must be a complex, Kähler manifold with holonomy contained in $SU(3)$: a Calabi-Yau manifold
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- anomaly cancellation: X and V must satisfy a topological condition relating their second Chern classes.
- Less obvious condition: the fundamental group of X has to be non-trivial to obtain the SM gauge group. This restricts the options for X to a few hundred examples.
- Many choices for V. Simplest choice: a sum of line bundles.

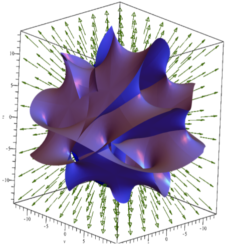
Computing the spectrum

- In heterotic models, 4d particle spectra (that is the multiplicities of the different multiplets present in the EFT) are determined by bundle-valued cohomology groups, $H^1(X, \wedge^k V)$.
- Example: $SU(5)$ line bundle models, $V = \bigoplus_{i=1}^{5} L_i$

multiplet	bundle	total number	required
$\mathbf{10}$	V	$\sum_{i=1}^{5} h^1(X, L_i)$	3
$\overline{\mathbf{10}}$	V^*	$\sum_i h^1(X, L_i^*)$	0
$\overline{\mathbf{5}}$	$\wedge^2 V$	$\sum_{i<j} h^1(X, L_i \otimes L_j)$	$3 + n_H$
$\mathbf{5}$	$\wedge^2 V^*$	$\sum_{i<j} h^1(X, L_i^* \otimes L_j^*)$	n_H
$\mathbf{1}$	$V \otimes V^*$	$\sum_{i,j} h^1(X, L_i \otimes L_j^*)$	

Line bundle cohomology formulae

topological data of (X, V)



global data:

cohomology groups

$h^{\bullet}(X, V)$

local data

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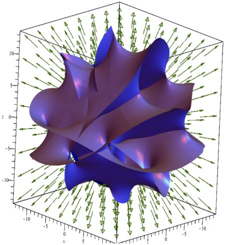

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Line bundle cohomology formulae

- The Hirzebruch-Riemann-Roch theorem gives

$$\chi(X, V) = \sum_{i=0}^{\dim(X)} (-1)^i h^i(X, V) = \int_X \mathrm{ch}(V) \cdot \mathrm{td}(X)$$

- Similar formulae have now been found for each cohomology

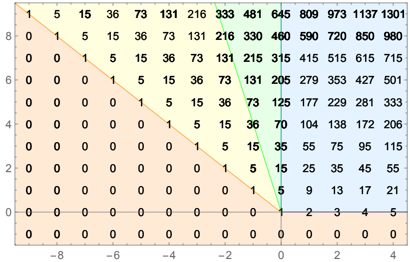
$$h^i(X, V) = \int_X \text{topological inv}(X, V)$$

when V is a sum of line bundles.

Line bundle cohomology formulae

- Initial hints from direct observation [AC, Lukas 18], [Larfors, Schneider 19] and ML [Klaewer, Schlechter 18], [Brodie, AC, Deen, Lukas 19]
- Theorems for surfaces, when X is toric, weak Fano and K3, for all line bundle cohomologies. [Brodie, AC 20], [Brodie, AC, Deen, Lukas 19]
- Good understanding of the zeroth cohomology for line bundles on Calabi-Yau threefolds. [Brodie, AC, Lukas 20]; [Brodie, AC, Gray, Lukas, Ruehle 21]
- Currently missing: understanding of the formulae arising for the middle cohomologies.
- For more details and references see 2112.12107, [Brodie, AC, Gray, Lukas, Ruehle].

A fairly innocent CY threefold example



$$X = \begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^4 \end{matrix} \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}^{2,86}$$

$$L = \mathcal{O}_X(k_1 D_1 + k_2 D_2)$$

$$\chi(X, L) = 2k_1(1 + k_2^2) + \frac{5}{6} k_2 (5 + k_2^2)$$

region in eff. cone	$h^0(X, L = \mathcal{O}_X(D = k_1 D_1 + k_2 D_2))$
$\mathcal{K}(X)$	$\chi(X, \mathcal{O}_X(D))$
$\overline{\mathcal{K}}(X') \setminus \{\mathcal{O}_X\}$	$\chi(X', \mathcal{O}_{X'}(D'))$
$\overline{\Sigma}$	$\chi\left(X', \mathcal{O}_{X'}\left(D' - \left\lceil \frac{D' \cdot \tilde{C}_2'}{\Gamma' \cdot \tilde{C}_2'} \right\rceil \Gamma'\right)\right)$
$k_1 \geq 0,\ k_2 = 0$	$\chi(\mathbb{P}^1, (D \cdot C_1) H_{\mathbb{P}^1})$

What are cohomology formulae useful for?

- Full information about the spectrum (not only the net number of chiral families), including the Higgs field and SM singlets.
- Bundle stability can be phrased in terms of cohomology.
- Hierarchy of Yukawa couplings.
- Absence of fast proton decay operators etc.

- All of these constraints can now be implemented in the search algorithms, since cohomology computations are virtually instantaneous.

Dealing with the immensity of the string landscape

- The existence of simple analytic formulae for bundle-valued cohomology greatly speeds up the analysis of heterotic string models
- However, this is not enough: the space of possible solutions is very large and systematic scans are unfeasible for $h^{1,1}(X) \gtrsim 7$.

AI methods for discrete optimisation

- Solution: intelligent methods for discrete optimisation, such as genetic algorithms (GAs) and reinforcement learning (RL).
- Essentially we are dealing with a bunch of diophantine equations (chiral index $= 3$, cohomology constraints, anomaly cancellation constraint, stability). The integers correspond to the topological data of X and V.
- Environment: space of potential (attempt) solutions, potentially very large.
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- Number of actual solutions: typically a tiny fraction of the environment.
- Both methods require a measure for how close an attempt solution is to an actual solution, call it intrinsic value function. This function should be sufficiently well-behaved for the methods to work (e.g. no abrupt changes from one state to nearby states).
- More details in Thomas Harvey's talk. [Abel, AC, Harvey, Lukas 21] (See also [Cole, Krippendorf, Schachner, Shiu 21] for model building in type II.)
- Important message: these methods (and in particular GAs) are extremely efficient in identifying good solutions.

The computation of physical Yukawa Couplings in string theory

Proceeds in two steps:

- the computation of holomorphic Yukawa couplings – can be accomplished using algebro-geometric methods
- the computation of physical Yukawa couplings requires knowledge of the normalisation factors for the matter fields, needed in order to have canonical kinetic terms – much more difficult

Physical Yukawa Couplings

- Need to know the matter field Kähler metric, which is proportional to the inner product

$$(\nu_i, \nu_j) = \int_X \bar{\nu}_i \wedge (H * \nu_j) = \int_X J \wedge J \wedge \bar{\nu}_i \wedge (H \nu_j)$$

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- Computing these quantities amounts to solving certain partial differential equations on curved backgrounds.
- Numerical methods using self-supervised machine learning are now available and the expectation is that within the next couple of years the calculation of physical Yukawa couplings will be possible. [Ashmore, He, Ovrut; Douglas; Anderson, Gerdes, Gray, Krippendorf, Raghuram, Ruehle; Jejjala, Pena, Mishra; Larfors, Lukas, Ruehle, Schneider...]
- These Yukawa couplings would be computed at specific values of the moduli. AI techniques will likely play an important role in figuring out the values of the moduli that lead to correct physical couplings.

Conclusion

- New tools: cohomology formulae, heuristic search methods, CY metrics/HYM connections.
- Good prospects in sight for string phenomenology!

Thank you for listening!